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Stochastic stability and robust control for sampled-data systems with Markovian jump parameters ☆

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Abstract

In this paper, the problems of stochastic stability and robust control for a class of uncertain sampled-data systems are studied. The systems consist of random jumping parameters described by finite-state semi-Markov process. Sufficient conditions for stochastic stability or exponential mean square stability of the systems are presented. The conditions for the existence of a sampled-data feedback control and a multirate sampled-data optimal control for the continuous-time uncertain Markovian jump systems are also obtained. The design procedure for robust multirate sampled-data control is formulated as linear matrix inequalities (LMIs), which can be solved efficiently by available software toolboxes. Finally, a numerical example is given to demonstrate the feasibility and effectiveness of the proposed techniques.

Keywords: Markovian jump system; Stochastic stability; Robust control; Sampled-data system; Linear matrix inequality

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1. Introduction

Modern industrial applications are encountered with numerous hybrid behavior of the processes. For example, any malfunction of sensors or actuators can cause a jumping behavior in process performance. This type of jumping behavior may be modelled as a Markov jump systems. Markovian jump system is a class of hybrid systems containing continuous-time or discrete-time dynamics which is governed by a stochastic discrete event. Thus, Markovian jump systems correspond to an important class of systems that are subject to abrupt process changes. The abrupt changes in the systems are discrete events and are assumed to be modelled by a Markov chain taking values in a finite value set. Practical motivations as well as many theoretical results for Markovian jump system can be found, for instance, in [1,3,6,10,11,14,32,33]. When a continuous-time Markovian jump system is controlled by digital control algorithms, the closed-loop system is referred to as a sampled-data system defined on the product space of a regular continuous-time space, a discrete-time space, and a sample space [7]. The study of sampled-data systems has scored a great success in the past several years (see [8,15,16,18,27,28,30,31] and reference therein for more details). Many existing works deal with linear sampled-data control problems such as H_2 and H_∞ problems, and servo problems using the frequency response approach [2,36], the lifting technique [5], and the L_2 approach [23,34]. In [12,24,34], the results of the robustness and stability of the uncertain sampled-data systems were presented. In [19,20], the authors considered H_2 and LQ robust sampled-data control problems under a unified framework. Multirate sampled-data control formulation has a strong practical justification. In [9], the authors pointed out several reasons for the use of a multirate sampling formulation. The first reason is that it may not always be possible to sample all signals at one single rate. Second, multirate sampled data formulation allows A/D and D/A devices asynchronous conversions and makes a better trade-off between performance and implementation cost [9,29]. Third, it makes implementation of a complex controller simpler and not adding burden to the finite memory. Moreover, multirate control formulation has a capability to realize arbitrary state feedback [17]. In [9,29], the authors considered multirate H_∞ and H_2 sampled-data control problem using a lifting technique. In [35], the authors parameterized H_∞ and H_2 multirate controller and employed the lifting technique as well. Robust H_∞ sampled-data control for linear systems was considered in [26]. However, to the best of authors' knowledge, to date, the problem of multirate sampled-data control for Markovian jump systems has not been investigated yet. The problem is interesting but also challenging, which motivates us to this study. In this paper, we first consider the stability problem of Markovian jump hybrid systems. Then we develop robust multirate sampled-data control and robust multirate sampled-data optimal control procedures for Markovian jump uncertain linear systems. The synthesis results are described as linear matrix inequalities which can be solved by available numerical software such as Matlab LMI toolbox.

The remaining of this paper is organized as follows. In Sections 2 and 3, we present some preliminary materials and a formulation of problems to be considered in this paper. The design procedures for robust multirate sampled-data control and multirate robust sampled-data optimal control for uncertain linear Markovian jump systems are developed in Sections 4 and 5. A numerical example is provided in Section 6. Concluding remarks are given in Section 7.

Notation. The notation used in this paper is fairly standard except where otherwise specified. Let T_s be the sampling period, with which all the states of the system are sampled simultaneously by ideal samplers. For a matrix M, $\bar{\sigma}(M)$ ($\underline{\sigma}(M)$) denotes the maximum (minimum) singular value of M. If M is real symmetric, $M < 0$ ($M > 0$) denotes M is negative (positive) definite. I stands for the identity matrix with compatible dimensions. $F(\theta^-)$ stands for the left limit of a function $F(\theta)$. $E\{\cdot\}$ represents the mathematical expectation operator.

2. Preliminary

Suppose $\mathbb{X}$ is a sample set. A (nonempty) collection $\mathbb{S}$ of subsets of $\mathbb{X}$ is a σ-algebra. A measure $\mathcal{P}:\mathbb{S} \to R_+$ is said to be a probability measure if $\mathcal{P}(\mathbb{X}) = 1$. We refer to $(\mathbb{X}, \mathbb{S}, \mathcal{P})$ as a probability space. Consider the following nonlinear stochastic hybrid system:

$$\dot{x}(t) = f\big(t, x(t), r(t)\big), \quad x\big(t_0^-\big) = x_0, \quad r\big(t_0^-\big) = r_0, \quad t_0 = 0, \tag{2.1}$$

$$x(t_k) = I_k\big(x\big(t_k^-\big), r\big(t_k^-\big)\big), \tag{2.2}$$

where $x \in \mathbb{R}^n$ denotes the state, and x_0 a fixed constant vector, respectively. Here it should be noted that $r(t)$ represents a discrete-state semi-Markov process with values in a finite set $\mathcal{J} = \{1, 2, \dots, s\}$. Its transition probabilities are defined as

$$\mathcal{P}\big\{r(t+s) = j \,\big|\, r(t) = i\big\} = q_{ij}(t, s).$$

For the sampling intervals $t \in [t_k, t_{k+1})$, $r(t)$ is the standard Markov process. Under the continuity condition $\lim_{s\to 0^+} q_{ij}(t, s) = \delta_{ij}$, where δ_{ij} is the Kronecker index, $\delta_{ij} = 1$, if $i = j$; 0, otherwise, then we have

$$\lim_{s\to 0^+} \frac{1 - q_{ii}(t, s)}{s} = -\pi_{ii} < \infty, \qquad \lim_{s\to 0^+} \frac{q_{ij}(t, s)}{s} = \pi_{ij} < \infty,$$

that is

$$\mathcal{P}\big\{r(t+s) = j \,\big|\, r(t) = i\big\} = \begin{cases} \pi_{ij}s + o(s), & i \neq j, \\ 1 + \pi_{ii}s + o(s), & i = j, \end{cases} \tag{2.3}$$

for $t \in [t_k, t_{k+1})$. On the other hand, we also need to define the transition probabilities from $[t_{k-1}, t_k)$ to $[t_k, t_{k+1})$ to describe the switching happening at the discrete-time $t = t_k^-$,

$$\mathcal{P}\big\{r(t_k) = j \,\big|\, r\big(t_k^-\big) = i\big\} = q_{ij}\big(t_k^-, 0^+\big) = \begin{cases} \pi_{ij}, & i \neq j, \\ 1 + \pi_{ii}, & i = j, \end{cases} \tag{2.4}$$

where π_{ij} is the jump rate from mode i to mode j that satisfies

$$\pi_{ij} \geqslant 0, \quad \forall i \neq j, \qquad \sum_{j=1, j\neq i}^{s} \pi_{ij} = -\pi_{ii}, \quad i \in \mathcal{J}. \tag{2.5}$$

Remark 2.1. It should be noted that this is a direct generalization of the traditional definitions of the transition rate for Markovian jump systems. In fact, for a process which is only defined at the sampling instants t_k^-, from (2.4) and (2.5), we know it is the standard discrete-time Markov process. On the other hand, for the continuous-time Markov process, from the continuity condition, the rate definition (2.3) can be defined over the whole time horizon, which is the exactly the same definition for the standard continuous-time Markov process. In this paper, the transition rate at the sampling instants is essential. In the industrial applications, especially, in networked-control systems [21], the controller (or the filter) will be switched or selected at the transition probability of a Markov process from the instants t_k^- to t_k, and be remained unchanged over the sampling interval. This definition of the transition probability in (2.3) and (2.4) describes the probabilistic behavior of the sampled-data systems.

For each $r(t)$, $f(\cdot,\cdot,r(t)):\mathbb{R}_+\times\mathbb{R}^n\to\mathbb{R}^n$ is continuous on interval $[t_k,t_{k+1})$ with left limit at t_k, bounded partial derivatives and locally Lipschitzian in x, and $I_k(\cdot,r(t_k)):\mathbb{R}^n\times\mathbb{R}^q\to\mathbb{R}^n$, $\{t_k\}$, $k=0,1,2,\ldots,$ are the sampling instants with $t_{k+1}-t_k=T_s$. From the theory of stochastic differential equations, we recall that system (2.1)–(2.2) admits a unique solution $x(t,x_0,r_0)\in(\mathbb{X},\mathbb{S},\mathcal{P})$ at time t, and initial distribution r_0 of r, $r_0\in\mathcal{J}$ [4]. The infinitesimal generator D acting on function $V(t,x,r)$ is given by

$$\begin{aligned}D^+V(t,x,i) &:= \lim_{h\to 0^+}\sup\frac{1}{h}\Big(E\big\{V\big(t+h,x(t+h),r(t+h)\big)\ \big|\ x(t)=x,\ r(t)=i\big\}\\ &\quad - V\big(t,x(t),i\big)\Big)\\ &= \frac{\partial}{\partial t}V(t,x,i)+\frac{\partial}{\partial x}V(t,x,i)f(t,x,i)+\sum_{j=1}^{s}\pi_{ij}V(t,x,j),\end{aligned}$$

for $t\in[t_k,t_{k+1})$.

Definition 2.1. For system (2.1)–(2.2), the equilibrium point 0 is

(1) stochastically stable, if for every initial state $x_0\in\mathbb{R}^n$ and initial distribution r_0 of $r(t)$,

$$E\left\{\int_0^\infty \big\|x(t,x_0,r_0)\big\|^2\,dt\right\}<\infty; \tag{2.6}$$

(2) exponentially mean square stable, if for every initial state $x_0\in\mathbb{R}^n$ and initial distribution r_0 of $r(t)$, there exist constants $\alpha>0$, $\beta>0$ such that

$$E\big\{\big\|x(t,x_0,r_0)\big\|^2\big\}\leqslant\alpha\|x_0\|^2e^{-\beta t},\quad \forall t\geqslant 0. \tag{2.7}$$

The stochastic stability properties of traditional continuous-time and discrete-time Markovian jump systems have received extensive attention [13,22]. In the following, we consider the stability problem of the hybrid system (2.1)–(2.2). The stochastic stability or exponential mean square stability for the hybrid Markovian system (2.1)–(2.2) will be then established.

Remark 2.2. Note that from [13], for standard Markovian jump linear continuous- or discrete-time systems, the concepts of stochastic stability and exponential stability are equivalent. By exactly the same techniques used in [13], it can be shown that for Markovian jump sampled-data system (2.1)–(2.2), the above mentioned two stability concepts are also equivalent.

Lemma 2.1. *A sufficient condition for stochastic stability or exponential stability of system* (2.1)–(2.2) *with initial condition x_0 and initial distribution r_0 is that there exists a Lyapunov functional candidate $V(t,x(t),r(t))$, for each $r(t)\in\mathcal{J}$, $V(\cdot,\cdot,r(t)):\mathbb{R}_+\times R^n\to\mathbb{R}_+$ being continuous on $[t_j,t_{j+1})$ with left limit at t_j and local Lipschitzian in x and strictly increasing positive function $a(\cdot)$, $b(\cdot)$ and $c(\cdot)$ with $a(0)=0$, $b(0)=0$, $c(0)=0$, such that*

$$b\big\|x(t,x_0,r_0)\big\|^2\leqslant V\big(t,x(t),i\big)\leqslant a\big\|x(t,x_0,r_0)\big\|^2 \tag{2.8}$$

satisfies

$$D^+V\big(t,x(t),i\big)\leqslant -c\big(\big\|x(t,x_0,r_0)\big\|\big), \tag{2.9}$$

for $t \in [t_j, t_{j+1})$, and

$$\sum_{j=kN_T}^{kN_T+l-1} E\bigl\{V\bigl(t_j, x(t_j), r(t_j)\bigr)\bigr\} \leqslant \chi_j\bigl(V\bigl(t_j^-, x\bigl(t_j^-\bigr), r\bigl(t_j^-\bigr)\bigr)\bigr), \tag{2.10}$$

for some integer $N_T \geqslant 0$, $k = 0, 1, \ldots,$ $l = 1, 2, \ldots, N_T$, where $\chi_j : \mathbb{R}_+ \to \mathbb{R}_+$ is a nondecreasing function with $\chi_j(s) \leqslant s$.

Proof. Considering the infinitesimal operator of the stochastic Lyapunov function $V(t, x(t), i)$ over the sampling interval, from (2.9), we have

$$\begin{aligned} D^+V\bigl(t, x(t), i\bigr) &\leqslant -c\bigl(\bigl\|x(t, x_0, r_0)\bigr\|\bigr) = -\frac{c(\|x(t, x_0, r_0)\|)}{V(t, x(t), i)} V\bigl(t, x(t), i\bigr) \\ &\leqslant -\alpha V\bigl(t, x(t), i\bigr), \end{aligned}$$

where the scalar $\alpha > 0$ is given by

$$\alpha = \min_x \frac{c(\|x(t, x_0, r_0)\|)}{a(\|x(t, x_0, r_0)\|)}.$$

By Dynkin's formula [25], for any $t \in [t_{\bar{k}}, t_{\bar{k}+1})$, and for some $\bar{k}$,

$$\begin{aligned} &E\left\{\sum_{j=0}^{\bar{k}-1} \int_{t_j}^{t_{j+1}} D^+V\bigl(s, x(s), r(s)\bigr)\, ds + \int_{t_{\bar{k}}}^{t} D^+V\bigl(s, x(s), r(s)\bigr)\, ds\right\} \\ &\quad= \sum_{j=0}^{\bar{k}-1} E\bigl\{V\bigl(t_{j+1}^-, x\bigl(t_{j+1}^-\bigr), r\bigl(t_{j+1}^-\bigr)\bigr)\bigr\} - V\bigl(t_j, x(t_j), r(t_j)\bigr) + E\bigl\{V\bigl(t, x(t), r(t)\bigr)\bigr\} \\ &\qquad - V\bigl(t_{\bar{k}}, x(t_{\bar{k}}), r(t_{\bar{k}})\bigr) \\ &\quad= E\bigl\{V\bigl(t, x(t), r(t)\bigr)\bigr\} - V\bigl(t_0^-, x_0, r_0\bigr) + \sum_{k=0}^{[\frac{\bar{k}}{N_T}]-1} \sum_{j=kN_T}^{(k+1)N_T-1} E\bigl\{V\bigl(t_j^-, x\bigl(t_j^-\bigr), r\bigl(t_j^-\bigr)\bigr)\bigr\} \\ &\qquad - V\bigl(t_j, x(t_j), r(t_j)\bigr) + \sum_{j=[\frac{\bar{k}}{N_T}]N_T}^{\bar{k}} E\bigl\{V\bigl(t_j^-, x\bigl(t_j^-\bigr), r\bigl(t_j^-\bigr)\bigr)\bigr\} - V\bigl(t_j, x(t_j), r(t_j)\bigr). \end{aligned}$$

Then we obtain from (2.10) that

$$\begin{aligned} &E\bigl\{V\bigl(t, x(t), r(t)\bigr)\bigr\} - V\bigl(t_0^-, x_0, r_0\bigr) \\ &\quad\leqslant E\left\{\sum_{j=0}^{\bar{k}-1} \int_{t_j}^{t_{j+1}} D^+V\bigl(s, x(s), r(s)\bigr)\, ds + \int_{t_{\bar{k}}}^{t} D^+V\bigl(s, x(s), r(s)\bigr)\, ds\right\} \\ &\quad\leqslant -\alpha E\left\{\sum_{j=0}^{\bar{k}-1} \int_{t_j}^{t_{j+1}} V\bigl(s, x(s), r(s)\bigr)\, ds + \int_{t_{\bar{k}}}^{t} V\bigl(s, x(s), r(s)\bigr)\, ds\right\} \\ &\quad= -\alpha E\left\{\int_0^t V\bigl(s, x(s), r(s)\bigr)\, ds\right\}. \end{aligned}$$

Above inequality implies that

$$\frac{d}{dt}E\{V(t,x(t),r(t))\} \leqslant -\alpha\frac{d}{dt}\int_0^t E\{V(s,x(s),r(s))\}\,ds = -\alpha E\{V(t,x(t),r(t))\},$$

and we have

$$E\{V(t,x(t),r(t))\} \leqslant V(t_0^-,x_0,r_0)e^{-\alpha t}.$$

The above inequality and (2.8) imply that the system (2.1)–(2.2) is exponentially mean square stable and therefore stochastically stable. This completes the proof of Lemma 1. □

3. Problem formulation

Consider the following uncertain linear Markovian jumping system:

$$\begin{aligned}&\dot{x}(t) = (A(r(t)) + \Delta A(r(t)))x(t) + (B(r(t)) + \Delta B(r(t)))u(t),\\ &x(t_0) = x_0, \quad r(0) = r_0,\end{aligned} \tag{3.1}$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are the state vector, and control input, respectively. $A(r(t))$, $B(r(t))$ are known constant matrices for each fixed $r(t) = i$ with appropriate dimensions. $\Delta A(r(t))$ and $\Delta B(r(t))$ are uncertain matrices satisfying:

$$\Delta A(r(t)) = H_1(r(t))\Delta(r(t))E_1(r(t)), \qquad \Delta B(r(t)) = H_2(r(t))\Delta(r(t))E_2(r(t)),$$

$\Delta(r(t))$ is unknown matrix function satisfying

$$\Delta(r(t))^T\Delta(r(t)) \leqslant I.$$

In the sequel, for simplicity, we will use A_i, B_i, H_{1i}, H_{2i}, E_{1i}, E_{2i}, Δ_i to represent the matrices $A(r(t))$, $B(r(t))$, $H_1(r(t))$, $H_2(r(t))$, $E_1(r(t))$, $E_2(r(t))$, $\Delta(r(t))$ when the mode $r(t) = i \in \mathcal{J}$.

Digital control is widely used in industrial process. A sampled-data control formulation allows a direct design of digital control. The present problem is to design a multi-rate sampled-data control for the system (3.1). We assume that all the states of plant are sampled by ideal samplers with the same sampling period T_s, and the control actions are taken with a time period T, and

$$N_T = \frac{T_s}{T}.$$

Here T_s is the frame period and N_T the input multiplicity. Moreover, the time-varying digital control signals are fed into the plant with ideal zero-order-holds. At every instant $t_k + lT$, its mechanism is described as

$$u(t) = \tilde{u}[l, r(t_k) \mid t_k], \quad \text{for } t \in [t_k + lT, t_k + (l+1)T), \tag{3.2}$$

for $l = 0, 1, 2, \ldots, N_T - 1$, where $\tilde{u}[l, r(t_k) \mid t_k]$ is defined as

$$\tilde{u}[l, r(t_k) \mid t_k] := F(l, r(t_k))x(t_k), \quad \text{for } l = 0, 1, 2, \ldots, N_T - 1, \tag{3.3}$$

with $F(l, r(t_k)) := F(lT, r(t_k))$ time-varying and periodic $F(l, r(t_k)) = F(l + N_T, r(t_k))$, that is,

$$F(lT + T_s, r(t_k)) = F(lT, r(t_k)).$$

In the frame period, the control gain is switched at $t = t_k + lT$. Here (3.3) represents a digital controller with gain $F(l, r(t_k)) =: F_{l,i}$. (3.2) means that the digital control is fed into the system via an ideal zero-order hold. Let $\tilde{x}(t) = (x^T(t), u^T(t))^T$, the closed-loop system (3.1) with (3.2)–(3.3) can be written as

$$\dot{\tilde{x}}(t) = (\bar{A}_i + \bar{H}_i \Delta_i \bar{E}_i)\tilde{x}(t), \quad t \in \big(t_k + lT, t_k + (l+1)T\big), \tag{3.4}$$

$$\tilde{x}(t_k + lT) = \tilde{A}\tilde{x}\big(t_k^- + lT\big) + \tilde{B}F_{l,i}\bar{C}_2\tilde{x}(t_k), \quad x(t_0) = x_0, \quad r(0) = r_0, \tag{3.5}$$

where

$$\bar{A}_i = \begin{bmatrix} A_i & B_i \\ 0 & 0 \end{bmatrix}, \qquad \bar{H}_i = \begin{bmatrix} H_{1i} & H_{2i} \\ 0 & 0 \end{bmatrix}, \qquad \bar{E}_i = \begin{bmatrix} E_{1i} & 0 \\ 0 & E_{2i} \end{bmatrix},$$
$$\tilde{A} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \qquad \tilde{B} = \begin{bmatrix} 0 \\ I \end{bmatrix}, \qquad \bar{C}_2 = \begin{bmatrix} I & 0 \end{bmatrix}, \qquad \mathbb{I} = \begin{bmatrix} I & 0 \end{bmatrix}^T.$$

Clearly, the closed-loop system composed of system (3.1) and controller (3.2)–(3.3) is a hybrid one. In the following, we will use Lemma 1 to develop two design procedures of multirate digital control and robust optimal digital control for the uncertain linear Markovian jump system of (3.1), respectively.

4. Robust digital control

By applying Lemma 2.1 to the closed-loop system (3.4)–(3.5), we can obtain the following result.

Theorem 4.1. *For system* (3.1), *if there exist matrix functions $X_i(t) = X_i(t + T_s) > 0$, for $t \geqslant 0$, defined as*

$$X_i(t) = X_i(l) + \frac{t - lT}{T}\big(X_i(l+1-0) - X_i(l)\big), \quad t \in \big[lT, (l+1)T\big), \quad i \in \mathcal{J}, \tag{4.1}$$

matrices $Z_i(l)$ and scalars $\xi_i > 0$ such that the following linear matrix inequalities hold:

$$\Phi_{i0} := \begin{bmatrix} S_{i0} & X_i(l)\bar{\Pi}^T & X_i(l)\bar{E}_i^T \\ \bar{\Pi}X_i(l) & -\bar{X}(l) & 0 \\ \bar{E}_i X_i(l) & 0 & -\xi_i I \end{bmatrix} < 0, \tag{4.2}$$

$$\Phi_{i1} := \begin{bmatrix} S_{i1} & X_i(l+1-0)\bar{\Pi}^T & X_i(l+1-0)\bar{E}_i^T \\ \bar{\Pi}X_i(l+1-0) & -\bar{X}(l+1-0) & 0 \\ \bar{E}_i X_i(l+1-0) & 0 & -\xi_i I \end{bmatrix} < 0, \tag{4.3}$$

and

$$\begin{bmatrix} -\bar{X}(0) + M_i & \bar{\Pi}\bar{C}_2^T\mathbb{Z}_i^T\mathbb{B}^T\tilde{\Pi}^T & \bar{\Pi}\tilde{B}Z_i(0) \\ \tilde{\Pi}\mathbb{B}\mathbb{Z}_i\bar{C}_2\bar{\Pi}^T & -\tilde{X} + \tilde{\Pi}\mathbb{A}\tilde{X}_i^-\mathbb{A}^T\tilde{\Pi}^T & \tilde{\Pi}\mathbb{B}\mathbb{Z}_i \\ Z_i^T(0)\tilde{B}^T\bar{\Pi}^T & \mathbb{Z}_i^T\mathbb{B}^T\tilde{\Pi}^T & -\mathbb{I}^T X_i(N_T - 0)\mathbb{I} \end{bmatrix} < 0, \tag{4.4}$$

where

$$S_{i0} = \frac{X_i(l) - X_i(l+1-0)}{T} + \bar{A}_i X_i(l) + X_i(l)\bar{A}_i^T + \xi_i \bar{H}_i \bar{H}_i^T,$$
$$S_{i1} = \frac{X_i(l) - X_i(l+1-0)}{T} + \bar{A}_i X_i(l+1-0) + X_i(l+1-0)\bar{A}_i^T + \xi_i \bar{H}_i \bar{H}_i^T,$$

$$
\begin{aligned}
&M_i = \bar{\Pi}\tilde{A}X_i(N_T - 0)\tilde{A}^T\bar{\Pi}^T + \bar{\Pi}\tilde{B}Z_i(0)\bar{C}_2\bar{\Pi}^T + \bar{\Pi}\bar{C}_2^T Z_i^T(0)\tilde{B}^T\bar{\Pi}^T,\\
&\bar{\Pi}^T = \begin{bmatrix}\pi_{i1}^{1/2}I & \cdots & \pi_{i(i-1)}^{1/2}I & (1+\pi_{ii})^{1/2}I & \pi_{i(i+1)}^{1/2}I & \cdots & \pi_{is}^{1/2}\end{bmatrix},\\
&\tilde{\Pi} = \operatorname{diag}\{\bar{\Pi}, \bar{\Pi}, \dots, \bar{\Pi}\},\\
&\bar{X}(l) = \operatorname{diag}\{X_1(l), \dots, X_{i-1}(l), X_i(l), X_{i+1}(l), \dots, X_s(l)\},\\
&\tilde{X} = \operatorname{diag}\{\bar{X}(1), \bar{X}(2), \dots, \bar{X}(N_T - 1)\},\\
&\tilde{X}_i^- = \operatorname{diag}\{X_i(1-0), X_i(2-0), \dots, X_i(N_T - 1 - 0)\},\\
&\mathbb{Z}_i = \begin{bmatrix}Z_i^T(1) & Z_i^T(2) & \cdots & Z_i^T(N_T - 1)\end{bmatrix}^T,\\
&\mathbb{A} = \operatorname{diag}(\tilde{A}, \tilde{A}, \dots, \tilde{A}),\\
&\mathbb{B} = \operatorname{diag}(\tilde{B}, \tilde{B}, \dots, \tilde{B}),
\end{aligned}
$$

then the control law (3.2)–(3.3) *with control gains* $F_{l,i} = Z_i(l)(\mathbb{I}^T X_i(N_T - 0)\mathbb{I})^{-1}$, $i \in \mathcal{J}$, *stochastically stabilizes system* (3.1).

Proof. Choose a Lyapunov functional candidate $V(t, \tilde{x}(t), r(t) = i) = \tilde{x}^T(t)P_i(t)\tilde{x}(t)$, where $P_i(t) = P_i(t + T_s) > 0$, $i = 1, 2, \dots, s$, are piecewise continuous functions defined on $t \geqslant 0$. Here take $P_i(t) = X_i^{-1}(t)$, where $X_i(t)$'s are the solutions of inequalities in (4.2)–(4.4). Let

$$
\mathbb{C}_2 = \begin{bmatrix}\bar{C}_2^T & \bar{C}_2^T & \cdots & \bar{C}_2^T\end{bmatrix}^T, \qquad \bar{F}_i = \{F_{1,i}^T \quad F_{2,i}^T \quad \cdots \quad F_{N_T-1,i}^T\}^T.
$$

For $t \in [t_k + lT, t_k + (l+1)T)$ and any scalars $\xi_i > 0$, we have

$$
\begin{aligned}
D^+V(t, \tilde{x}(t), i) &= \lim_{\epsilon \to 0^+} \frac{1}{\epsilon}\big[E\{V(t+\epsilon, \tilde{x}(t+\epsilon), r(t+\epsilon)) \mid r(t) = i\} - V(t, \tilde{x}, i)\big]\\
&\leqslant \tilde{x}^T(t)\big(D^+P_i(t) + P_i(t)\bar{A}_i + \bar{A}_i^T P_i(t) + \bar{\Pi}^T\bar{X}^{-1}(t)\bar{\Pi}\\
&\quad + \xi_i P_i(t)\bar{H}_i\bar{H}_i^T P_i(t) + \xi_i^{-1}\bar{E}_i^T\bar{E}_i\big)\tilde{x}(t).
\end{aligned}
$$

From the construction in (4.1), the periodic positive definite matrix function $X_i(t)$ is convex on the interval $[lT, (l+1)T)$ and is a convex combination of $X_i(l)$ and $X_i(l+1-0)$. As the linear combination of (4.2) and (4.3) is negative definite, we have

$$
\begin{aligned}
&\frac{X_i(l) - X_i(l+1-0)}{T} + \bar{A}_i X_i(t) + X_i(t)\bar{A}_i^T + X_i(t)\bar{\Pi}^T\bar{X}^{-1}(t)\bar{\Pi}X_i(t)\\
&\quad + \xi_i\bar{H}_i\bar{H}_i^T + \xi_i^{-1}X_i(t)\bar{E}_i^T\bar{E}_i X_i(t) < 0.
\end{aligned}
$$

Pre- and post-multiplying above inequality by $P_i(t)$ and noting $P_i(t)X_i(t) = I$, we obtain

$$
D^+V(t, \tilde{x}(t), i) < 0, \quad \text{for all } i \in \mathcal{J}. \tag{4.5}
$$

At $t = t_k + lT$, from the multi-rate control (3.2)–(3.3), letting

$$
\Theta(t_k) := \big(\tilde{x}^T(t_k^- + T), \dots, \tilde{x}^T(t_k^- + (N_T - 1)T)\big)^T,
$$

we have

$$
\sum_{l=0}^{N_T-1}\big[E\{V(t_k + lT, \tilde{x}, r) \mid \tilde{x}(t_k^- + lT), r(t_k^- + lT) = i\} - V(t_k^- + lT, \tilde{x}, i)\big]
$$

$$
\begin{aligned}
&= \sum_{l=1}^{N_T-1} \bigl\{\tilde{x}^T\bigl(t_k^- + lT\bigr)\tilde{A}^T\bar{\Pi}^T\bar{X}^{-1}(l)\bar{\Pi}\tilde{A}\tilde{x}\bigl(t_k^- + lT\bigr) \\
&\quad + 2\tilde{x}^T\bigl(t_k^- + lT\bigr)\tilde{A}^T\bar{\Pi}^T\bar{X}^{-1}(l)\bar{\Pi}\tilde{B}F_{l,i}\bar{C}_2\tilde{x}(t_k) \\
&\quad + \tilde{x}^T(t_k)\bar{C}_2^T F_{l,i}^T\tilde{B}^T\bar{\Pi}^T\bar{X}^{-1}(l)\bar{\Pi}\tilde{B}F_{l,i}\bar{C}_2\tilde{x}(t_k) \\
&\quad - \tilde{x}^T\bigl(t_k^- + lT\bigr)X_i^{-1}(l-0)\tilde{x}\bigl(t_k^- + lT\bigr)\bigr\} \\
&\quad + \tilde{x}^T(t_k)\bigl\{-X_i^{-1}(N_T-0) + (\tilde{A}+\tilde{B}F_{0,i}\bar{C}_2)^T\bar{\Pi}^T\bar{X}^{-1}(0)\bar{\Pi}(\tilde{A}+\tilde{B}F_{0,i}\bar{C}_2)\bigr\}\tilde{x}(t_k) \\
&= \Theta^T(t_k)\bigl(-\bigl(\tilde{X}_i^-\bigr)^{-1} + \mathbb{A}^T\tilde{\Pi}^T\tilde{X}^{-1}\tilde{\Pi}\mathbb{A}\bigr)\Theta(t_k) + 2\Theta^T(t_k)\mathbb{A}^T\tilde{\Pi}^T\tilde{X}^{-1}\tilde{\Pi}\mathbb{B}\bar{F}_i\bar{C}_2\tilde{x}(t_k) \\
&\quad + \tilde{x}^T(t_k)\bigl\{-X_i^{-1}(N_T-0) + (\tilde{A}+\tilde{B}F_{0,i}\bar{C}_2)^T\bar{\Pi}^T\bar{X}^{-1}(0)\bar{\Pi}(\tilde{A}+\tilde{B}F_{0,i}\bar{C}_2) \\
&\quad + \bar{C}_2^T\bar{F}_i^T\mathbb{B}^T\tilde{\Pi}^T\tilde{X}^{-1}\tilde{\Pi}\mathbb{B}\bar{F}_i\bar{C}_2\bigr\}\tilde{x}(t_k) \\
&= \bigl[\tilde{x}^T(t_k) \quad \Theta^T(t_k)\bigr]\Xi\bigl[\tilde{x}^T(t_k) \quad \Theta^T(t_k)\bigr]^T,
\end{aligned}
$$

where

$$
\begin{aligned}
\Xi &= \begin{bmatrix} -X_i^{-1}(N_T-0) & 0 \\ 0 & -(\tilde{X}_i^-)^{-1} \end{bmatrix} \\
&\quad + \begin{bmatrix} (\tilde{A}+\tilde{B}F_{0,i}\bar{C}_2)^T\bar{\Pi}^T\bar{X}^{-1}(0)\bar{\Pi}(\tilde{A}+\tilde{B}F_{0,i}\bar{C}_2) + \bar{C}_2^T\bar{F}_i^T\mathbb{B}^T\tilde{\Pi}^T\tilde{X}^{-1}\tilde{\Pi}\mathbb{B}\bar{F}_i\bar{C}_2 & * \\ \mathbb{A}^T\tilde{\Pi}^T\tilde{X}^{-1}\tilde{\Pi}\mathbb{B}\bar{F}_i\bar{C}_2 & \mathbb{A}^T\tilde{\Pi}^T\tilde{X}^{-1}\tilde{\Pi}\mathbb{A} \end{bmatrix}.
\end{aligned}
$$

For $[\tilde{x}^T(t_k)\ \Theta^T(t_k)]^T \neq 0$, if

$$\Xi < 0, \tag{4.6}$$

then

$$\sum_{l=0}^{N_T-1}\bigl[E\bigl\{V(t_k+lT,\tilde{x},r) \bigm| \tilde{x}\bigl(t_k^-+lT\bigr), r\bigl(t_k^-+lT\bigr)=i\bigr\} - V\bigl(t_k^-+lT,\tilde{x},i\bigr)\bigr] < 0. \tag{4.7}$$

If there exists a solution to (4.2)–(4.3) and (4.4), then (4.1) shows that the matrix $X_i(t)$ satisfies

$$\min\bigl\{\underline{\sigma}\bigl(X_i(l)\bigr), \underline{\sigma}\bigl(X_i(l+1-0)\bigr)\bigr\}I \leqslant X_i(t) \leqslant \max\bigl\{\bar{\sigma}\bigl(X_i(l)\bigr), \bar{\sigma}\bigl(X_i(l+1-0)\bigr)\bigr\}I,$$

for $t \geqslant 0$, which implies $P_i(t)$ is bounded and the Lyapunov function satisfies bounds of the type in (2.8) of Lemma 2.1. By Lemma 2.1 with conditions (4.5) and (4.7), the closed-loop system (3.4)–(3.5) is stochastically stable. From (4.6), we obtain

$$
\begin{bmatrix}
-X_i^{-1}(N_T-0) & 0 & (\tilde{A}+\tilde{B}F_{0,i}\bar{C}_2)^T\bar{\Pi}^T & \bar{C}_2^T\bar{F}_i^T\mathbb{B}^T\tilde{\Pi}^T \\
0 & -(\tilde{X}_i^-)^{-1} & 0 & \mathbb{A}^T\tilde{\Pi}^T \\
\bar{\Pi}(\tilde{A}+\tilde{B}F_{0,i}\bar{C}_2) & 0 & -\bar{X}(0) & 0 \\
\tilde{\Pi}\mathbb{B}\bar{F}_i\bar{C}_2 & \tilde{\Pi}\mathbb{A} & 0 & -\tilde{X}
\end{bmatrix} < 0,
$$

which is again equivalent to

$$
\begin{aligned}
&\begin{bmatrix} -\bar{X}(0) & 0 \\ 0 & -\tilde{X} \end{bmatrix} \\
&\quad + \begin{bmatrix} \bar{\Pi}(\tilde{A}+\tilde{B}F_{0,i}\bar{C}_2) & 0 \\ \tilde{\Pi}\mathbb{B}\bar{F}_i\bar{C}_2 & \tilde{\Pi}\mathbb{A} \end{bmatrix}\begin{bmatrix} X_i(N_T-0) & 0 \\ 0 & \tilde{X}_i^- \end{bmatrix}
\end{aligned}
$$

$$\times \begin{bmatrix} (\tilde{A}+\tilde{B}F_{0,i}\bar{C}_2)^T\bar{\Pi}^T & \bar{C}_2^T\bar{F}_i^T\mathbb{B}^T\tilde{\Pi}^T \\ 0 & \mathbb{A}^T\tilde{\Pi}^T \end{bmatrix} < 0.$$

Expand the second group on the left side of the above inequality

$$\begin{aligned}
&\begin{bmatrix} \bar{\Pi}(\tilde{A}+\tilde{B}F_{0,i}\bar{C}_2) & 0 \\ \tilde{\Pi}\mathbb{B}\bar{F}_i\bar{C}_2 & \tilde{\Pi}\mathbb{A} \end{bmatrix} \begin{bmatrix} X_i(N_T-0) & 0 \\ 0 & \tilde{X}_i^- \end{bmatrix} \begin{bmatrix} (\tilde{A}+\tilde{B}F_{0,i}\bar{C}_2)^T\bar{\Pi}^T & \bar{C}_2^T\bar{F}_i^T\mathbb{B}^T\tilde{\Pi}^T \\ 0 & \mathbb{A}^T\tilde{\Pi}^T \end{bmatrix} \\
&\quad = \begin{bmatrix} \bar{\Pi}\tilde{A}X_i(N_T-0)\tilde{A}^T\bar{\Pi}^T & 0 \\ 0 & \tilde{\Pi}\mathbb{A}\tilde{X}_i^-\mathbb{A}^T\tilde{\Pi}^T \end{bmatrix} + \tilde{M} + \tilde{M}^T \\
&\qquad + \begin{bmatrix} \bar{\Pi}\tilde{B}F_{0,i}\bar{C}_2 & 0 \\ \tilde{\Pi}\mathbb{B}\bar{F}_i\bar{C}_2 & 0 \end{bmatrix} \left[\begin{bmatrix} X_i(N_T-0) & 0 \\ 0 & \tilde{X}_i^- \end{bmatrix} \right] \begin{bmatrix} \bar{C}_2^T F_{0,i}^T\tilde{B}^T\bar{\Pi}^T & \bar{C}_2^T\bar{F}_i^T\mathbb{B}^T\tilde{\Pi}^T \\ 0 & 0 \end{bmatrix},
\end{aligned}$$

where

$$\begin{aligned}
\tilde{M} &= \begin{bmatrix} \bar{\Pi}\tilde{A} & 0 \\ 0 & \tilde{\Pi}\mathbb{A} \end{bmatrix} \begin{bmatrix} X_i(N_T-0) & 0 \\ 0 & \tilde{X}_i^- \end{bmatrix} \begin{bmatrix} \bar{C}_2^T F_{0,i}^T\tilde{B}^T\bar{\Pi}^T & \bar{C}_2^T\bar{F}_i^T\mathbb{B}^T\bar{\Pi}^T \\ 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} \bar{\Pi}\tilde{A}X_i(N_T-0)\bar{C}_2^T F_{0,i}^T\tilde{B}^T\bar{\Pi}^T & \bar{\Pi}\tilde{A}X_i(N_T-0)\bar{C}_2^T\bar{F}_i^T\mathbb{B}^T\bar{\Pi}^T \\ 0 & 0 \end{bmatrix}.
\end{aligned}$$

Noting $\tilde{A}X_i(N_T-0)\bar{C}_2^T = \bar{C}_2^T(\mathbb{I}^T X_i(N_T-0)\mathbb{I})$, $\bar{C}_2 X_i(N_T-0)\bar{C}_2^T = \mathbb{I}^T X_i(N_T-0)\mathbb{I}$ and letting $F_{l,i}\mathbb{I}^T X_i(N_T-0)\mathbb{I} := Z_i(l)$, we obtain the condition expressed in (4.4). These complete the proof. □

5. Robust LQ digital control

In this section, we consider the sampled-data optimal control problem associated with (3.1) and minimization of an upper bound of the quadratic cost functional

$$J = \sup_{\Delta:\ \|\Delta\|<1} E\left\{ \int_0^\infty \left(x^T(t)Q_i x(t) + u^T(t)R_i u(t)\right) dt \right\}, \tag{5.1}$$

where $Q_i \geqslant 0$, $R_i > 0$. Let $\tilde{Q}_i := \operatorname{diag}(Q_i, R_i)$.

Theorem 5.1. *For system* (3.1)*, if there exist matrix functions $X_i(t) = X_i(t+T_s) > 0$, for $t \geqslant 0$ defined as* (4.1)*, matrices $U_i > 0$, $Z_i(l)$ and scalars $\xi_i > 0$, then control $F_{l,i} = Z_i(l)(\mathbb{I}^T \times X_i(N_T-0)\mathbb{I})^{-1}$, $i \in \mathcal{J}$, which stochastically stabilizes the system in* (3.1) *and minimizes an upper bound of the cost in* (5.1)*, is a solution of the following optimization problem*:

$$\inf_{Z_i,\xi_i,X_i} \operatorname{trace} U_i, \tag{5.2}$$

subject to the following linear matrix inequalities:

$$\begin{bmatrix} U_i & I \\ I & X_i(N_T-0) \end{bmatrix} > 0, \tag{5.3}$$

and

$$\Phi_{i0} + \operatorname{diag}(\tilde{Q}_i, \quad 0, \quad \cdots, \quad 0) < 0, \tag{5.4}$$

$$\Phi_{i1} + \operatorname{diag}(\tilde{Q}_i, \quad 0, \quad \cdots, \quad 0) < 0, \tag{5.5}$$

plus condition (4.4). *Furthermore, the performance cost satisfies*

$$J < \inf_{Z_i, \xi_i, X_i} \operatorname{trace} U_i. \tag{5.6}$$

Proof. Choose a Lyapunov functional candidate $V(t, \tilde{x}(t), r(t) = i) = \tilde{x}^T(t) P_i(t) \tilde{x}(t)$, where $P_i(t) = P_i(t + T_s) > 0$, $i = 1, 2, \ldots, s$, are piecewise continuous functions defined on $t \geqslant 0$. Here, take $P_i(t) = X_i^{-1}(t)$ where $X_i(t)$, $i = 1, 2, \ldots,$ are the solutions of inequalities (4.4) and (5.4)–(5.5). Similar to the proof of Theorem 4.1, for $t \in (t_k + lT, t_k + (l+1)T)$, and any $\xi_i > 0$, from (5.4)–(5.5), we have

$$\begin{aligned} &D^+ P_i(t) + P_i(t)\bar{A}_i + \bar{A}_i^T P_i(t) + \bar{\Pi}^T \bar{X}^{-1}(t)\bar{\Pi} + \xi_i P_i(t)\bar{H}_i \bar{H}_i^T P_i(t) + \xi_i^{-1} \bar{E}_i^T \bar{E}_i \\ &\quad < -\tilde{Q}_i. \end{aligned}$$

Then we have

$$\begin{aligned} J(\tilde{x}_0) &= \sup_{\Delta} E\left\{ \lim_{N\to\infty} \sum_{k=0}^{N} \sum_{j=0}^{N_T-1} \int_{t_k+jT}^{t_k+(j+1)T} \left(\tilde{x}^T(t)\tilde{Q}_i\tilde{x}(t) + D^+V\big(t, \tilde{x}(t)\big) \right.\right. \\ &\qquad \left.\left. - D^+V\big(t, \tilde{x}(t)\big)\right) dt \right\} \\ &< \inf_{\xi_i, P_i} \left(- \lim_{N\to\infty} E\left\{ \sum_{k=0}^{N} \sum_{j=0}^{N_T-1} \int_{t_k+jT}^{t_k+(j+1)T} D^+V\big(t, \tilde{x}(t), i\big)\, dt \right\} \right) \\ &= \inf_{\xi_i, P_i} \left(V\big(t_0^-, \tilde{x}(t_0), r_0\big) \right. \\ &\qquad \left. + \lim_{N\to\infty} E\left\{ \sum_{k=0}^{N} \sum_{j=0}^{N_T-1} \Big(V(t_k + jT, \tilde{x}) - V\big(t_k^- + jT, \tilde{x}\big)\Big) - V\big(t_k^- + N_T T, \tilde{x}\big) \right\} \right). \end{aligned}$$

Moreover, (4.4) leads to

$$J < \inf_{F_i, \xi_i, P_i} V\big(t_0^-, \tilde{x}(t_0), r_0\big) = \inf_{F_i, \xi_i, P_i} \tilde{x}^T\big(t_0^-\big) P_i(0^-) \tilde{x}\big(t_0^-\big).$$

Let U_i be chosen as

$$U_i > P_i(0^-),$$

then

$$J < \inf_{F_i, \xi_i, P_i} \operatorname{trace} U_i,$$

which directly leads to (5.2)–(5.3). Moreover, by Lemma 2.1 and following the same arguments as in the proof of Theorem 4.1, it can be shown that closed-loop system (3.4)–(3.5) is stochastically stable. This completes the proof. □

6. Numerical examples

In this section, we use a numerical example to demonstrate the design procedure of robust sampled-data optimal control for the Markovian jump system (3.1).

Example 6.1. Consider a sampled-data control for the following system:

$$A_1 = \begin{bmatrix} 1.5 & 1 \\ 0 & -1.5 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -1.5 & 0 \\ 1 & 1.5 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad B_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad A_4 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \qquad B_4 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$A_5 = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}, \qquad B_5 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

with the transition rate $\pi_{11}(t) = -1$, $\pi_{12}(t) = 1$, $\pi_{33}(t) = -1$, $\pi_{34}(t) = 1$, for $t \in [t_k, t_{k+1})$, $\pi_{22}(t) = -1$, $\pi_{23}(t) = 0.5$, $\pi_{25}(t) = 0.5$, $\pi_{43}(t) = 1$, $\pi_{44}(t) = -1$, for $t = t_k^-$, the other transition rate $\pi_{ij}(t) = 0$, and $R_1 = R_2 = R_3 = R_4 = R_5 = 1$, $Q_1 = Q_2 = Q_3 = Q_4 = Q_5 = I$. The sampling period is selected as $T_s = 0.1s$.

Theorem 5.1 formulated an optimization problem which can be solved by the command *mincx* of LMITool for Matlab. Selecting different input multiplicity N_T, we can obtain the following performance indexes: $J = 1.2198 \times 10^{-5}$, 4.1298×10^{-5}, 5.6341×10^{-5}, for $N_T = 2$, $N_T = 3$, $N_T = 4$, respectively.

Simulation result shows that the performance index depends on the input multiplicity N_T. Bigger N_T results higher upper bound of J. In fact, what we computed is an upper bound of the accumulated effect excited by impulses in the interval $(0, T_s]$ at every controller switching instants [20].

7. Conclusion

The multi-rate sampled-data control problems for uncertain systems with Markovian jump parameters were considered in this paper. To facilitate control synthesis, based on Lyapunov technique, a sufficient condition to ensure the stochastic stability for nonlinear stochastic hybrid systems was established. Based on this stability condition, a robust digital controller and a robust digital LQ controller were considered to stabilize and minimize an LQ cost function, respectively, for the uncertain systems with Markovian jump parameters, while their control design procedures are formulated as an LMI feasibility problem and an optimization problem subject to LMIs, respectively. Finally, a numerical example demonstrated the feasibility of the proposed design techniques.

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